Axisymmetric convective states of a binary liquid confined in a vertical cylinder

E. Millour[a,1] , E. Tric[b] and G. Labrosse[a]
[a] Université Paris-Sud, Laboratoire d'Informatique pour la Mécanique et les Sciences de l'Ingénieur LIMSI-CNRS, BP 133, 91403 ORSAY CEDEX, FRANCE
[b] Université Nice Sophia Antipolis, Laboratoire Géosciences AZUR, 250 rue Albert Einstein, 06560 VALBONNE, FRANCE

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Abstract

Axisymmetric convection of a binary liquid confined in a vertical cylinder (of radius R and height $h = 2R$) heated from below is numerically investigated. Two sets of boundary conditions are considered, relevant to different physical configurations. We first give the ranges (in Rayleigh number Ra) of the oscillatory and steady convective states for a given mixture and show that a change in its characteristics (an increase of the Prandlt number Pr value from 1 to 10) can significantly alter the overall behaviour of the system, namely the concealement of the time dependent state. We then compare, for both sets of boundary conditions and high Ra values, binary and pure fluid convective flows and exhibit to what extent their behaviours differ.

1 Geometrical configuration and system parameters

The equations ruling binary liquid convection, once rendered nondimensional and under the Oberbek-Boussinesq hypothesis, include four parameters. Among them is the Rayleigh number: $Ra = \frac{\alpha \Delta T g h^3}{\kappa\nu}$, where h is the height of the cylinder, g the gravitationnal acceleration, α the thermal expansion coefficient, κ and ν the thermal and momentum diffusivities and ΔT the imposed temperature difference between bottom and top plates. The three other parameters entirely depend on the nature of the considered liquid mixture: the Prandlt $Pr = \frac{\nu}{\kappa}$ and Lewis $Le = \frac{\kappa_s}{\kappa}$ (where κ_s is the mass diffusivity) numbers and the separation ratio $\Psi = \frac{\beta}{\alpha}\frac{k_T}{T_0}$ (T_0, β, and k_T standing for the mean temperature, mass expansion and thermodiffusion coefficients). In the present contribution, the values of these last two numbers are set to $Le = 0.1$ and $\Psi = -0.2$.
The following boundary conditions are considered:
- Imposed temperature on horizontal plates and thermally insulated side wall,
- No mass flux through any boundary,
- No-slip kinematic condition on top and bottom boundaries.

As for the kinematic condition on the side boundary, we shall investigate both cases of either a no-slip or slip-free. the first (NS) refers to a liquid enclosed in a cylindrical box whereas the last (SF) is a first approximation of a (straight and surface tension free) liquid bridge [1].

[1]speaker and author for correspondence: millour@limsi.fr

2 Binary Liquid convective states at low Ra

2.1 $Pr = 1$ Case

Computations have been run, for both configurations with $Pr = 1$. The resulting domains over which different convective states are obtained are summarized in Figures 1 and 2.

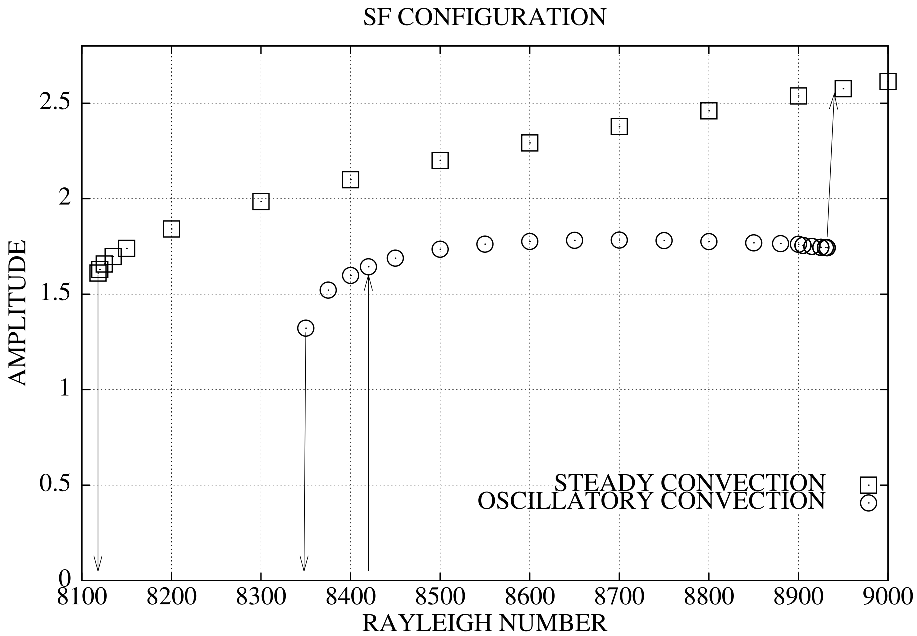

Figure 1: *Observed convective states. The given amplitudes are those of the radial velocity at a given node (null amplitude hence depicts the conductive state).*

Albeit for different threshold values, the global behaviour is the same in both NS and SF configurations: once the value of Ra exedes a critical value, the conductive state undergoes a (subcritical) Hopf bifurcation that leads to oscillatory convection. The way these time-dependent states then evolve happens to be different wether the NS or SF case is considered but are not in the scope of the present contribution and will therefore not be further detailed. In both configurations, once the thermal driving becomes too strong, oscillatory behaviours abruptly end and the resulting flow is a steady convective one. Since those three states are not directly connected, there are common ranges in Ra over which they exist which yield the possibility of as many hysteretical behaviours (indicated by arrows in Figures 1 and 2).

2.2 $Pr = 10$ Case

If Pr is now set to 10, a major change in the global behaviour of the system is observed (again occuring for both NS and SF cases): The Hopf bifurcations ($Ra_H^{NS} \sim 15600$ and $Ra_H^{SF} \sim 8550$) of the conductive states do not lead to oscillatory convection but to steady one instead. Once the stationnary flow

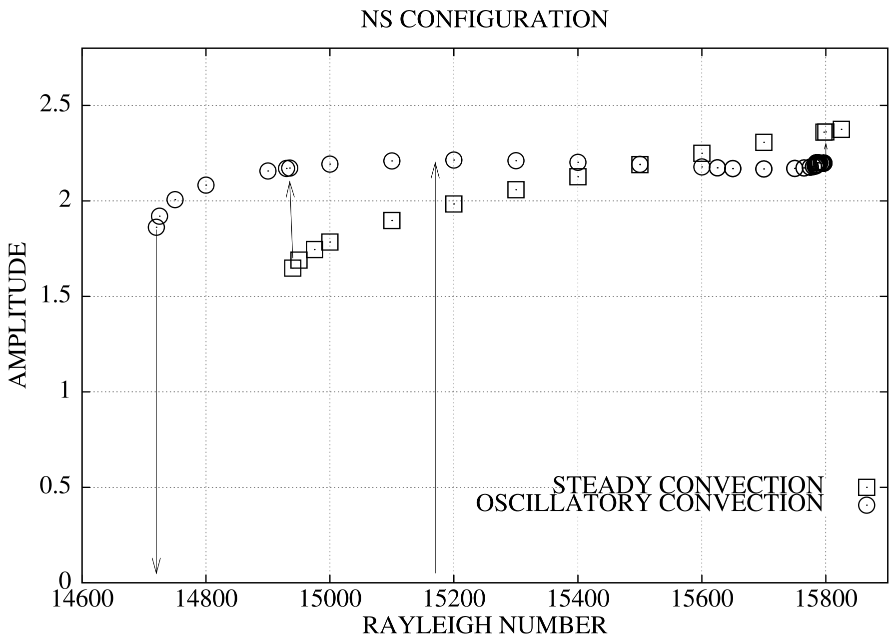

Figure 2: *As in Figure 1. The fact that branches cross is merely due to the choice of the monitored amplitude.*

is obtained, with decreasing Ra one follows the steady states branch until the thresholds ($Ra_{sta}^{NS} \sim 14100$ and $Ra_{sta}^{SF} \sim 7850$ that lead back to the conductive regime are reached; time-dependent states are nowhere to be found! –Ref (Pr=10,Le=0.01,psi=-0.25) [2] Transition SOC-TW directe–

These last states do however exist and were reached by using a different strategy: a $Pr = 1$ oscillatory flows was used as an initial condition and both Ra and Pr values were gradually changed. Having reached the oscillatory branches and explored the domains ($Ra^{NS} \in [14425, 14875]$ and $Ra^{SF} \in [8175, 8400]$) over which they dwell explains why the could not be observed before: these branches lie at Ra values lower than Ra_H and greater than Ra_{sta}, in regions where both steady convective and conductive states are strongly stable.

3 Comparison with pure fluid steady convection

For both sets of boundary conditions, computations relative to pure ($Pr = 1$, $\Psi = 0$) fluid were run. In pure fluids, the conductive state undergoes a supercritical pitchfork bifurcation to steady convection and we shall here only compare binary and pure fluid steady states.

3.1 SF configuration

Figure 3 shows, for the SF configuration, the global kinetic energy of the obtained flows.

$$\mathcal{E}_c = \iiint_{\Omega} v.v d\Sigma$$

It is clear that at high Ra, the fact that the liquid be binary or not has little influence on its convective behaviour, as already mentioned in [3], [4] and [5].

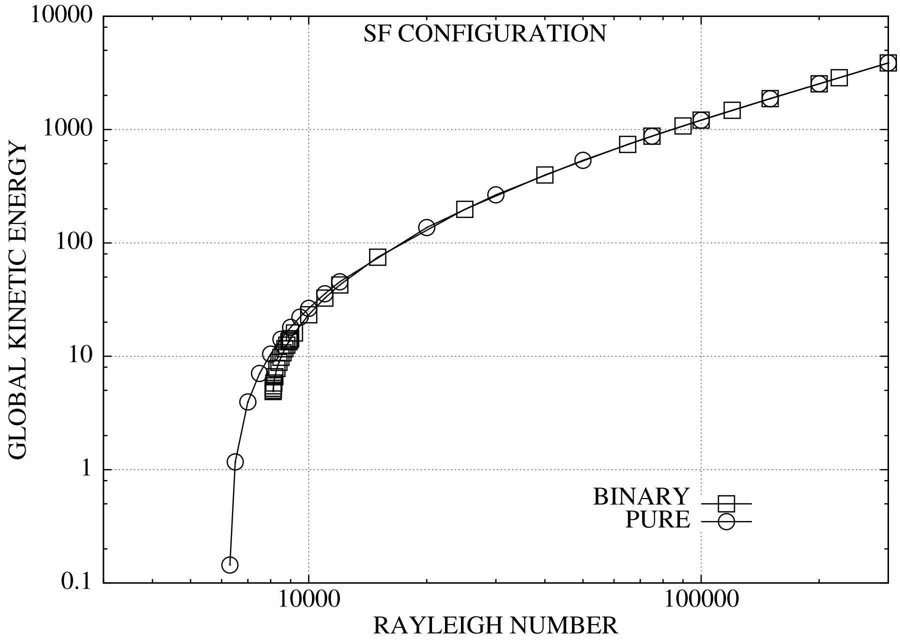

Figure 3: *Global kinetic energy of pure fluid and binary liquid steady states.*

3.2 NS Configuration

The NS configuration is found to be much more complex than the SF one; Indeed, as summarized in Figure, there are multiple possible convective states for most Ra values.

In the NS configuration (figure 4), there are two possible kinds of flow, consisting of either one or two (stacked) convective cells. We will not detail this last regime here but rather focus on the unicellular state that is born at low Ra values and has a structure similar to the one obtained in the SF configuration. For that flow (figure 4), the global kinetic energy curves relative to pure and binary liquids do collapse for $Ra \in [20000, 75000]$ but then part; Moreover, the pure fluid steady state vanishes at $Ra \sim 85000$ whereas the binary liquid state loses its stability at $Ra \sim 105000$. It is also worth mentionning that beyond that threshold, in the pure fluid case, the flow becomes oscillatory and quite similar (in structure and frequency) to the one found for the binary liquid at low Ra.

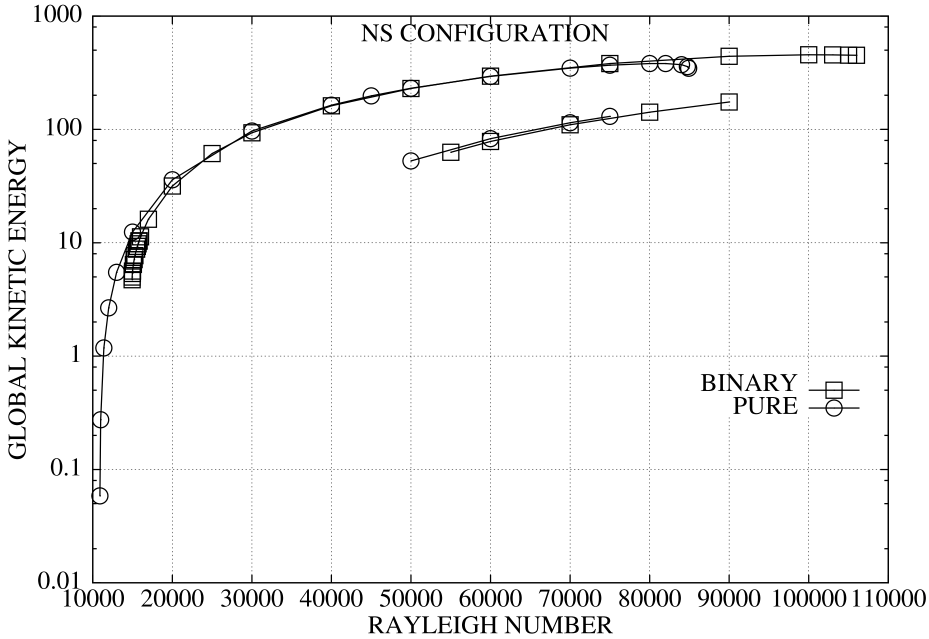

Figure 4: *Global kinetic energy of the steady states; Upper and lower branches respectively correspond to one and two (stacked) cells flows*

All these differences clearly imply that binary and pure liquid steady convection can indeed be almost identical, provided the considered states are far from their bifurcations, and that whatever the value of Ra may be, the bifurcations types and values of the flows remain strongly tied to the (binary or not) nature of the considered liquid.

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