Bifurcations undergone by the oscillatory convective states of an enclosed binary liquid

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Abstract

The axisymmetric convective states of a binary liquid enclosed in a vertical cylinder heated from below are determined by pseudo-spectral numerical integration. In order to gain some insight on the constraints that nearby boundaries can exert on the convective flows, three aspect ratios (radius over height of the cylinder), as well as two types of lateral kinematic boundary conditions (either no-slip or free-slip) are investigated. The oscillatory flows that occur are found to undergo a large variety of local and global bifurcations, the occurrences of which depend on both aspect ratio values and boundary conditions.

1 Introduction

Since the first investigations of Bénard and Rayleigh, convection in horizontal fluid layers heated from below is a problem that has been extensively studied in the context of pattern formation, instabilities and dynamical behavior of nonlinear systems. About 30 years ago, the case of binary mixtures [5] in the same straightforward configuration was investigated and turned out to yield additional complex spatiotemporal behavior, which can moreover arise (in contradistinction to pure fluids in the same conditions) as the quiescent state turns unstable. In binary liquid mixtures, solute mass fraction and temperature gradients are coupled by the Soret effect. Consequently, a homogeneous mixture, once subjected to a thermal gradient, separates in composition. According to the sign of the Soret coefficient, the solute (which we take to be the heaviest of the two components) migrates towards the warmer or colder part of the container. The resulting Soret-driven mass fraction gradient thus induces a solutal buoyancy that works with or against the thermal one. Both sign and amplitude of the Soret coupling are accounted for in the dimensionless separation ratio ψ. When ψ is negative, thermal and solutal buoyancies

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compete and the interplay between the two leads to the aforementioned dynamical behaviors.

An impressive number of studies have been published (see for instance [1],[4] and references therein) on this system, mainly focused on horizontally infinite or very extended layers. In such systems, endwalls are sufficiently distant so that a preferential horizontal direction for wave propagation is generated and hence favors 2D flows. However, for most practical systems where mixtures are expected to convect, the ratios between the enclosure's extents are more modest. In such geometries, the presence of nearby walls can be expected to exert a significant influence on the convective flows' dynamics, as indicated by experimental results (e.g: [2, 3, 6]).

The case of thermal convection of a binary liquid enclosed in a cylindrical cell of small aspect ratio Γ (radius over height ratio) is here tackled. Three aspect ratios are considered: $\Gamma = 1/2$, 1 and 2, as well as two types of boundary conditions for the velocity on the circumference of the cylinder, either no-slip or free-slip. The former simply represents the presence of a rigid wall, whereas the latter is a crude approximation (capillary effects being discarded) of a straight free surface and the modeled system then corresponds to a liquid bridge [7].

2 Ruling equations and geometrical settings

The usual Oberbeck-Boussinesq equations, Soret effect included, are considered. Using the height h of the layer, the thermal diffusion velocity over h, the imposed temperature difference ΔT and the Soret-induced mass fraction difference ΔC of the quiescent state as reference scales leads to the following set of dimensionless equations:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}.\nabla)\mathbf{v} = -\nabla p + Ra\, Pr\,(\theta + \Psi\gamma)\,\mathbf{e_z} + Pr\,\nabla^2\mathbf{v}, \tag{1}$$

$$\nabla.\mathbf{v} = 0, \tag{2}$$

$$\frac{\partial \theta}{\partial t} + (\mathbf{v}.\nabla)\theta = \mathbf{v}.\mathbf{e_z} + \nabla^2\theta, \tag{3}$$

$$\frac{\partial \gamma}{\partial t} + (\mathbf{v}.\nabla)\gamma = \mathbf{v}.\mathbf{e_z} + Le\,(\nabla^2\gamma - \nabla^2\theta), \tag{4}$$

where $\mathbf{v} = u\mathbf{e_r} + w\mathbf{e_z}$ is the velocity, $\mathbf{e_r}$ and $\mathbf{e_z}$ respectively being the radial and upward unit vectors. p, θ and γ denote departures from the static pressure, temperature and mass fraction profiles. Ra, Pr and Le are the usual Rayleigh, Prandtl and Lewis numbers and Ψ is the separation ratio between the solutal and thermal contributions to the density. The last three parameters are constant for a given fluid and here set to $Pr = 1$, $Le = 0.1$, $\Psi = -0.2$, leaving Ra as the main parameter of this study.

Temperatures are imposed on the horizontal walls, along with lateral thermal insulation and impermeability at all boundaries. Top and bottom boundary conditions are:

$$\theta = \frac{\partial \theta}{\partial z} - \frac{\partial \gamma}{\partial z} = u = w = 0 \ \text{ for } \ z = \pm\frac{1}{2}.$$

Denoting the aspect ratio $\Gamma = R/h$, where R is the radius of the cylinder, the lateral conditions on the scalar fields are:

$$\frac{\partial \theta}{\partial r} = \frac{\partial \gamma}{\partial r} = 0 \text{ for } r = \Gamma.$$

Two sets of kinematic lateral boundary conditions are considered:

$$\text{(a) No-slip:} \quad u = w = 0 \text{ for } \ r = \Gamma,$$

or

$$\text{(b) Slip-free:} \quad u = \frac{\partial w}{\partial r} = 0 \text{ for } \ r = \Gamma.$$

In all that follows, these will be referred to as (a) NS and (b) FS configurations.

3 Results

3.1 General features of the branches of solutions

The typical evolution of the system (for all cases investigated) with increasing Ra is the following:

1. The liquid remains motionless if the imposed temperature difference ΔT (or equivalently the value of Ra) is too small.

2. When Ra becomes greater than a threshold value Ra_H, the quiescent state turns unstable via a subcritical Hopf bifurcation leading to oscillatory regimes.

3. Time-dependent motion exists over a limited range in Ra. Decreasing Ra below a threshold value Ra_{SNo} (which is lower than Ra_H) brings the system back to the motionless state. Increasing the value of Ra beyond another threshold value Ra_{SOC} triggers a transition to stationary convection.

4. The later transition is, in the present case (in contradistinction to the smoother continuous one obtained in extended systems, e.g: [4]) always yields hysteresis: steady state solutions remain stable for decreasing Ra down to yet another threshold value Ra_{SNs}. Below the later, the system evolves towards either (depending on the case studied i.e: the set of boundary conditions and aspect ratio Γ considered) the oscillatory or quiescent state.

3.2 Specific features of the oscillatory solutions

A detailed investigation of the oscillatory branches of solutions in all six configurations ($\Gamma = 1/2, 1$ and 2 for both NS and FS boundary conditions) has been lead. It revealed a great deal of features specific to each configuration that can be summarized as follows:

- In all cases, the oscillatory solutions that stem from the subcritical Hopf bifurcation of the quiescent state share a given temporal symmetry T. Apart for the $\Gamma = 2$, NS configuration (where it holds over the whole of the oscillatory domain), this symmetry eventually breaks down, leading to other types of (nonetheless still time-dependent) solutions.

- In the FS configurations and for all Γ, increasing Ra leads to a complex sequence of bifurcations: temporal symmetry breaking is followed by a period doubling route to chaos, itself followed by crisis induced intermittency as the two attractors (born as T is broken) merge, statistically restoring temporal symmetry. This final attractor then vanishes in a boundary-induced crisis that leads to stationary convection.

- In the NS configuration, such route to chaos does not arise. For $\Gamma = 2$, the oscillatory branch is found to end in a generalized subcritical Hopf bifurcation. For $\Gamma = 1/2$ and 1, another sequence of bifurcations leading to steady convection emerges: temporal symmetry breaking first arises (as in the FS configuration) and the branch of oscillatory solutions thus born ends in a homoclinic bifurcation as the associated limit cycle collides with an unstable steady state solution.

4 Conclusions

The investigation of the axisymmetrical states of a binary liquid enclosed in a vertical cylinder has shown that the constraints exerted on the convective flows by nearby side boundaries can lead to significant changes in the system's dynamical behavior.

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